

Exam 2 Extra Credit
Due Monday, November 26

Answer the following questions on a separate *lined* notebook page. Make sure to use full sentences and correct grammar. You will be graded on the accuracy of your work, as well as the clarity of your argument and your use of the English language. *Plagiarism will not be tolerated!* Each question is worth four points to be added to your score on Exam 2. When working on Question n you are free to assume all previous questions; i.e. Questions 1, 2, \dots , $n - 1$, are true even if you have not completed those previous questions. For example, when answering Question 5, you may assume that Question 2 is true even if you were unable to prove it.

1. Explain why Theorems 2-4 of the Section 2.1 and 2.6 Worksheet do not give us any information about whether or not an Euler path exists in the case of a simple, connected graph consisting of *exactly one* vertex of odd degree.
2. Prove that an Euler path exists for every simple, connected graph consisting of *exactly one* vertex of odd degree.
3. Give a short (1-2 line) proof that an Euler path exists for every simple, connected graph consisting of *exactly one* vertex of odd degree. (**Hint.** Use Theorem 1 of the Section 2.1 and 2.6 Worksheet.)
4. Explain why doing Question 3 correctly earns you eight points instead of only four points.
5. Show that Question 2 above together with Theorems 2-4 of the Section 2.1 and 2.6 Worksheet imply the following theorem:

Theorem 1 *A simple, connected graph G has an Euler path if and only if there are at most two odd degree vertices in G .*